INTRODUCTION

1. The necessity of the dissertation

The Twin Rotor MIMO System (TRMS_Twin Rotor Multi-Input Multi-Output System) is an aerodynamic experiment set, whose movements simulated like those of a helicopter. TRMS is a typical nonlinear control object with multi-input multi-output, significant coupling, parameter uncertainty, and effects disturbance. Therefore, many domestic and foreign researchers have taken TRMS as the research object to develop and test new control methods, especially for the problem of accurate position control. Although each project has achieved results based on the criteria and methods of establishing the control system, TRMS is still a big challenge for researchers to apply new control algorithms to improve tracking trajectory performance. Therefore, the author has chosen the dissertation topic "Research on Desinging Trajectory Tracking Controller for Twin Rotor MIMO System" to make more significant new scientific contributions in theoretical research as well as applicability in practice for this nonlinear object class.

2. The goals and tasks of the dissertation

* Goals of the dissertation:

Research to design an exact linearized feedback controller combines with the uncertainty and disturbance identifiers for the electromechanical system, which is described by the Euler-Lagrange model in general and the TRMS in particular.

For accomplish this goal, the dissertation sets out the main tasks:

- Study on the technique to install an exact linearized feedback controller for the Euler-Lagrange system class, when it has the exact model.

- Supply a disturbance identification function as well as the additional controller for this disturbance compensation to the main controller. This work extends the applicability and the performance of the exact linearized controller, including the Euler-Lagrange system classes with inaccurate models.

3. Object and research scope of the dissertation

* Object:

Object class is represented by the bilinear Euler-Lagrange model with uncertainty in general and TRMS in particular. For TRMS, this is a system with almost full simulation function of a helicopter and belongs to the group of mechatronic systems with the general Euler-Lagrange model.

* Research scope:

- Study of designing control method for an incorrect bilinear Euler-Lagrange model with system uncertainty functions, so that the outputs (the joint variables) of the system asymptotic tracking follow the desired reference trajectory.

- Apply above the method to TRMS. The proposed controller quality is tested through simulation and experimentation.

4. Research methods

To achieve the set goals, the dissertation used following research methods:

- Theoretical research: Analysis and syndissertation of knowledge about mathematical model errorsof the Euler-Lagrange system in general and the TRMS system in particular, the source of model deviations. From that will propose an appropriate control method, in particular here is the adaptive control method to compensate for model errors combine with the feedback accurate linear control method.

- Simulation research: Using the Matlab-Simulink tool to simulate to verify the theoretical judgments and proposed algorithms by the dissertation.

- Researching results are proved by experiment in real conditions, i.e., to do experiments to appreciate the stability adapted performance of the proposed controller on the physical test table of the TRMS.

5. Contributions of the dissertation

* *Dissertation has detail contributions such as:*

- Establishing the trajectory tracking controller for the bilinear Euler-Lagrange system has an exact model. Dissertation proved the

tracking controller is the stability and asymptotic stability in both case certainty and uncertainty factors;

- Establishing the adaptive trajectory tracking controller for the uncertainty bilinear electromechanical system by the exact linearized controller and the uncertain identifier based on time receding optimization.

* Scientific significance of the dissertation:

- From the controller according to PID law of the equipment supplier for TRMS, the dissertation has designed an uncertain compensation controller based on time receding optimization to achieve high accuracy on reference trajectories;

- Contributing a small part to the diverse development of automatic control theory for nonlinear systems with uncertainty parameters and effects disturbance

* Practical significance of the dissertation:

- Diversifying control methods for TRMS model, applied in graduate and postgraduate training of the university;

- The results of this research will apply for flying devices with complex aerodynamic form.

6. Structure of the dissertation

The main content of the dissertation has four chapters and the conclusion, includes the research issues:

Chapter 1 presents an overview of the modeling and the available control methods of the TRMS. Thereby, clarifying the urgency of dissertation in proposing TRMS appropriate control method to improve control system performance.

Chapter 2 establishes the exact linearized controller for the TRMS when it has an exact model.

Chapter 3 establishes the compensating control for function uncertainty based on time receding optimization. Then, combine with the accurateness linearized controller to be the stability adapted controller for the TRMS.

Finally, in chapter 4, the performance of the exact linearized controller combined with the disturbance compensation identifier for the model's uncertainty will be verified by the TRMS physical experimental table in real conditions.

CHAPTER 1
OVERVIEW OF TRMS - MODELING AND METHODS OF CONTROLS

1.1 Modeling of TRMS
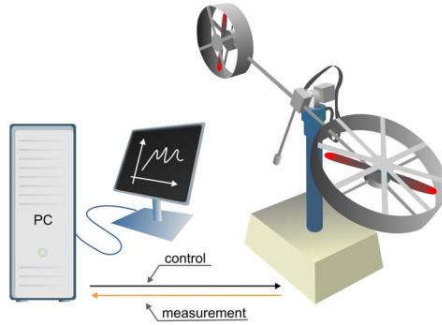### *1.1.1 TRMS physical structure*

Figure 1.1: TRMS physical structure [8]

Figure 1.1 shows the TRMS physical structure [8]. This is a laboratory for experimental. Figure 1.2 below describes in detail the electrical and mechanical relationship in TRMS.
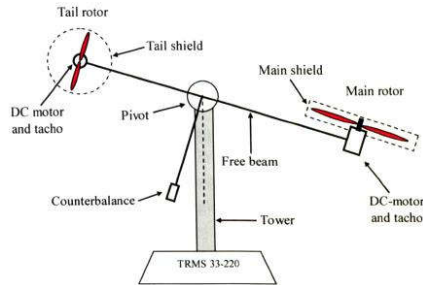

Fig 1.2: Electro-mechanical structure TRMS [9]-[10]

1.1.2 Modeling by the theoretical method
1.1.2.1 Euler-Lagrange equation

The Euler-Lagrange equation of a mechanical system is structured as follows [11]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \underline{\dot{q}}}\right)^T - \left(\frac{\partial L}{\partial \underline{q}}\right)^T = F\underline{\tau} \tag{1.1}$$

Lagrange equations

Apply the Euler-Lagrange equation (1.1) to TRMS [9], [10]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\alpha_h}\right) - \frac{\partial L}{\partial \alpha_h} = \sum_i M_{ih} \tag{1.27}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\alpha_v}\right) - \frac{\partial L}{\partial \alpha_v} = \sum_i M_{iv} \tag{1.28}$$

where:

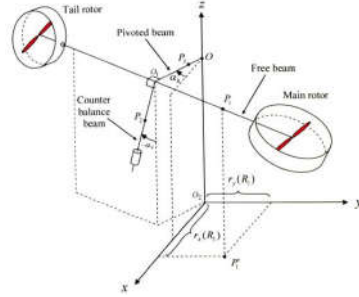$$L = \sum_{i=1}^{3} W_{di} - \sum_{i=1}^{3} W_{ti} \tag{1.29}$$

Figure 1.3: TRMS [10]

Perform the two equations above together in vector form, this yields:

$$\begin{bmatrix} J_1 \cos^2\alpha_v + J_2 \sin^2\alpha_v + h^2\left(m_{T_1}+m_{T_2}\right) + J_3 & h\left(m_{T_1}l_{T_1}\sin\alpha_v - m_{T_2}l_{T_2}\cos\alpha_v\right) \\ h\left(m_{T_1}l_{T_1}\sin\alpha_v - m_{T_2}l_{T_2}\cos\alpha_v\right) & \left(J_1+J_2\right) \end{bmatrix}\begin{bmatrix}\ddot\alpha_h \\ \ddot\alpha_v\end{bmatrix}$$
$$+\begin{bmatrix} h\left(m_{T_1}l_{T_1}\cos\alpha_v + m_{T_2}l_{T_2}\sin\alpha_v\right)\dot\alpha_v^2 + 2\left(J_2-J_1\right)\dot\alpha_h\dot\alpha_v\sin\alpha_v\cos\alpha_v \\ \dot\alpha_h^2\left(J_1-J_2\right)\sin\alpha_v\cos\alpha_v + g\left(m_{T_1}l_{T_1}\cos\alpha_v + m_{T_2}l_{T_2}\sin\alpha_v\right) \end{bmatrix} = \begin{bmatrix}\sum_i M_{ih} \\ \sum_i M_{iv}\end{bmatrix} \tag{1.30}$$

where:

$$\sum_i M_{ih} = M_{prop.h} - M_{fric,h} - M_{cable} + k_m\dot\omega_v\cos\alpha_v \tag{1.33}$$

$$\sum_i M_{iv} = M_{prop.v} - M_{fric.v} + k_t\dot\omega_h + M_{gyro} \tag{1.34}$$

α_h : Yaw angle [rad], α_v : Pitch angle [rad], ω_h : Rotational velocity of the tail rotor [rad/s], ω_h : Rotational velocity of the tail rotor [rad/s]

1.1.2.2. Model parameter identification

The detailed steps of the above identification procedure for TRMS were in [9] and [10].
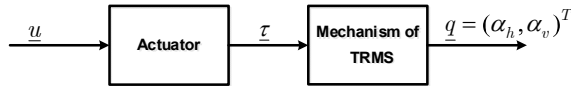
Figure 1.6: Overall block structure of the TRMS physical system

1.2 Previous control methods and overview of the related studies

Euler-Lagrange's equation for a general electromechanical system [11],[12],[13],[15]: $M(\underline{q})\ddot{\underline{q}} + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}} + \underline{g}(\underline{q}) = F\underline{\tau} + \underline{n}(t)$ (1.40)

Applying the TRMS model described by equation (1.30) into equation (1.40) is given the parameters:

$$\underline{\tau} = \begin{pmatrix} M_{prop.h} \\ M_{prop.v} \end{pmatrix}, \ \underline{q} = \begin{pmatrix} \alpha_h \\ \alpha_v \end{pmatrix}, \ M(\underline{q}) = \big(m_{ij}(\underline{q})\big), \ C(\underline{q},\dot{\underline{q}}) = \big(c_{ij}(\underline{q},\dot{\underline{q}})\big), \ i,j = 1,2$$

$$F = I_{2\times 2}, \ \underline{g}(\underline{q}) = \Big(0 \ , \ g(m_{T_1} l_{T_1} \cos\alpha_v + m_{T_2} l_{T_2} \sin\alpha_v) \Big)^T$$

$$\underline{n}(t) = \begin{pmatrix} -M_{fric.h} - M_{cable} + k_m \dot{\omega}_v \cos\alpha_v \\ -M_{fric.v} + k_t \dot{\omega}_h + M_{gyro} \end{pmatrix}$$

$$m_{11}(\underline{q}) = J_1 \cos^2\alpha_v + J_2 \sin^2\alpha_v + h^2(m_{T_1} + m_{T_2}) + J_3$$

$$m_{12}(\underline{q}) = m_{21}(\underline{q}) = h\big(m_{T_1} l_{T_1} \sin\alpha_v - m_{T_2} l_{T_2} \cos\alpha_v\big), \ m_{22}(\underline{q}) = J_1 + J_2,$$

$$c_{11}(\underline{q},\dot{\underline{q}}) = 2\dot{\alpha}_v (J_2 - J_1) \sin\alpha_v \cos\alpha_v$$

$$c_{12}(\underline{q},\dot{\underline{q}}) = \dot{\alpha}_v h\big(m_{T_1} l_{T_1} \cos\alpha_v + m_{T_2} l_{T_2} \sin\alpha_v\big)$$

$$c_{21}(\underline{q},\dot{\underline{q}}) = \dot{\alpha}_h (J_1 - J_2) \sin\alpha_v \cos\alpha_v \ \text{ và } \ c_{22} = 0. \tag{1.42}$$
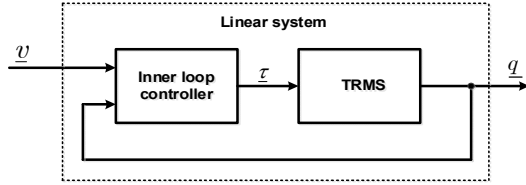
1.2.1 Linear control

Figure 1.7: Linearization using the feedback controller

Assuming that the electromechanical model (1.40), or the TRMS described by the math model (1.30), (1.42) satisfies:

- Absolutely accurate;
- Fully actuated;
- And without any uncertainty, i.e., n(t) = 0, then a controller (inner loop) that makes the feedback system linear, this yields:

$$\underline{\tau} = M(\underline{q})\underline{v} + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}} + \underline{g}(\underline{q}). \tag{1.43}$$

We can see, immediately that with the internal loop controller (1.43) given above, the feedback system in Figure 1.7 will become linear (2nd-degree integral and decoupling):

$$M(\underline{q})\underline{v} = M(\underline{q})\ddot{\underline{q}} \ \text{ hay } \ \underline{v} = \ddot{\underline{q}}, \tag{1.44}$$

For $M(\underline{q})$ is the inverse matrix (positively determined). Moreover, we also see that with the controller (1.43), the system (1.44) is not only linear but also decoupling. With such an internal loop controller

(1.43), the next control problem is only linear system control, decoupling (1.44) by a linearizing controller (in the outer loop) so that the whole closed-loop system obtain the desired tracking performance, i.e., let its' output $\underline{q}$ asymptotically to the reference vector $\underline{r}$.

1.2.1.1 PID control
1.2.1.2 Optimal control of LQR and LQG
1.2.2 Nonlinear control
1.2.2.1 Sliding control
1.2.2.2 Adaptive control
1.2.2.3 Fuzzy control
1.2.2.4 Neural network control
1.2.2.6 Predictive control and receding horizon control
1.3 Conclusions
In chapter 1, the dissertation presented an overview of the Twin Rotor MIMO system, analyzing the mathematical model based on Euler-Lagrange, referring to the nonlinear problems of the object researching as TRMS. Hence, it can see that TRMS is a nonlinear MIMO system with two inputs / two outputs, significant coupling, uncertainty parameters, and disturbance effects. Through overall analysis and evaluation of the research of domestic and foreign authors with research directions on controlling motion trajectory of TRMS thereby clarifying the urgency of the dissertation, proposed establishing the trajectory tracking adaptive controller for the uncertainty bilinear electromechanical system based on the linearized controller, and the uncertainty identifier based on time receding optimization of the model deviation applies to TRMS. This controller takes advantage of the existing controllers and avoids their disadvantages. Therefore, the dissertation has set out the requirements:
- Still using the reliable advantage of the continuous linear control methods (don't need to discrete the model for controller design);
- Add on linear control methods in the continuous time-domain, or at least those linearized methods, adaptability, and robustness to nonlinear uncertainties.

CHAPTER 2
EXACT LINEARIZED CONTROL FOR THE TRMS WITH EXACT MODEL

In this chapter, the dissertation will seek to establish the trajectory tracking controller for the electromechanical system with the bilinear Euler-Lagrange model: $M(\underline{q})\ddot{\underline{q}} + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}} = F\left[\underline{u} + \underline{d}(\underline{q},t)\right]$ (2.1) instead of the original structure (1.40)

It could realize that if compared with (1.40) this model (2.1) is equivalent if the unknown functions $\underline{n}(t)$, as well as the friction force vector and the gravitational acceleration $\underline{g}(\underline{q})$ of (1.40) are assumed to belong to the image space of F:

$$\underline{n}(t) = F\underline{n}(t)' \; and \; \underline{g}(\underline{q}) = F\underline{g}(\underline{q})' \qquad (2.2)$$

Then there is the relationship between $\underline{n}(t), \underline{g}(\underline{q})$ of (1.40) and

disturbance $\underline{d}(\underline{q},t)$ of (2.1): $\underline{d}(\underline{q},t) = \underline{n}(t)' - \underline{g}(\underline{q})'$ (2.3)

2.1 Original method: Gravity compensation control

The gravitational compensation control method is a design method for the reference trajectory tracking controller, denoted $\underline{r}$, for the joint variable $\underline{q}$ for electromechanical system class (1.40) full actuator without any uncertainties (called an exact model system):

$$M(\underline{q})\ddot{\underline{q}} + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}} + \underline{g}(\underline{q}) = \underline{u} \qquad (2.4)$$

In which, $\underline{u} = \underline{\tau}$, means that the actuator is assumed to be an ideal transformation of physical values.
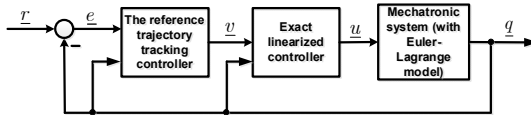

Figure 2.1: The cascade structure of the gravity compensation controller [4]
2.1.1 Exact linearization using feedback signals

Feedback linearization controller (inner loop control) for mechatronic systems (2.4) the same controller was known before (1.43):

$$\underline{u} = M(\underline{q})\underline{v} + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}} + \underline{g}(\underline{q}) \qquad (2.5)$$

and with it the closed system in the inner loop, becomes linear: $\ddot{\underline{q}} = \underline{v}$ (2.6)

Moreover, the above linear system (2.6) is also coupling, including n the SISO subsystem (single input single output) of the 2nd degree integral form:

$\ddot{q}_i = v_i, \ i = 1, 2, \ \ldots, n$

, where n is the number of variables (also the number of input signals).

2.1.2. Tracking trajectory using outer loop control

The outer loop controller is defined as [4]:

$$\underline{v} = \frac{d^2 \underline{r}}{dt^2} + K_1 \underline{e} + K_2 \frac{d\underline{e}}{dt} \text{ với } \underline{e} = \underline{r} - \underline{q} \quad (2.7)$$

, where $\underline{r}[rad]$, $\underline{e}[rad]$, $K_1[rad/s^2]$, $K_2[rad/s]$

Two matrices K_1, K_2 are chosen such that: $\Phi = \begin{bmatrix} 0 & 0 \\ -K_1 & -K_2 \end{bmatrix}$ (2.8)

, is the Hurwitz matrix. With the above outer loop controller, the exact linearized system (2.6) will become:

$$\ddot{\underline{q}} = \underline{v} = \frac{d^2 \underline{r}}{dt^2} + K_1 \underline{e} + K_2 \frac{d\underline{e}}{dt} \text{ or } \underline{0} = \frac{d^2 \underline{e}}{dt^2} + K_1 \underline{e} + K_2 \frac{d\underline{e}}{dt},$$

It will become $\begin{pmatrix} \dot{\underline{e}} \\ \ddot{\underline{e}} \end{pmatrix} = \Phi \begin{pmatrix} \underline{e} \\ \dot{\underline{e}} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \underline{e} \\ \dot{\underline{e}} \end{pmatrix} = \exp(\Phi t) \begin{pmatrix} \underline{e}(0) \\ \dot{\underline{e}}(0) \end{pmatrix}$ (2.9)

Thus, $\underline{e} \to \underline{0}$ and $\dot{\underline{e}} \to \underline{0}$, because the Φ is the Hurwitz matrix.

2.1.3 Combination controller

Putting the two controllers (2.5) and (2.7) together is obtained:

$$\underline{u} = M(\underline{q})\left[\ddot{\underline{r}} + K_1 \underline{e} + K_2 \dot{\underline{e}}\right] + C(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} + g(\underline{q}) \quad (2.10)$$

with $\underline{e} = \underline{r} - \underline{q}$

2.2 The proposed method for the bilinear Euler-Lagrange system with the exact model

$$M(\underline{q})\ddot{\underline{q}} + C(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} = \underline{u} \quad (2.11)$$

2.2.1 Controller for reference trajectory tracking

Modify the combination controller (2.10) so that it matches the bilinear Euler-Lagrange form so that in equation (2.11), this yields:

$$\underline{u} = M(\underline{q})\left[\ddot{\underline{r}} + K_1 \underline{e} + K_2 \dot{\underline{e}}\right] + C(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} \text{ với } \underline{e} = \underline{r} - \underline{q} \quad (2.12)$$
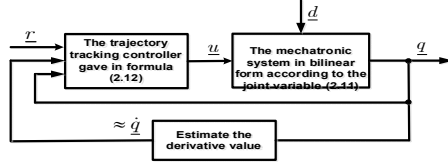
Figure 2.2: Reference trajectory tracking control for the bilinear system according to joint variable

Theorem 1: For matrices K_1, K_2: $K_1 = diag(k_{1i})$, $K_2 = diag(k_{2i})$ with $k_{2i}^2 > k_{1i} > 0$ (2.16) then the controller will make the output of the bilinear system according to the joint variable, described by the exact model (2.11), asymptotic to the reference signal $\underline{r}$:

Proof:

Sign of two matrices:

$$Q = \begin{pmatrix} 2K_1^2 & 0 \\ 0 & 2(K_2^2 - K_1) \end{pmatrix} and \ P = \begin{pmatrix} 2K_1K_2 & K_1 \\ K_1 & K_2 \end{pmatrix} \qquad (2.17)$$

With the conditions in (2.16), the P and the Q are the definite positive symmetrical matrices. Next, the Φ is given by (2.8), we have:

$$\Phi^T P + P\Phi = \begin{pmatrix} 0 & -K_1 \\ I & -K_2 \end{pmatrix}\begin{pmatrix} 2K_1K_2 & K_1 \\ K_1 & K_2 \end{pmatrix} + \begin{pmatrix} 2K_1K_2 & K_1 \\ K_1 & K_2 \end{pmatrix}\begin{pmatrix} 0 & I \\ -K_1 & -K_2 \end{pmatrix} \qquad (2.18)$$

$$= -\begin{pmatrix} 2K_1^2 & 0 \\ 0 & 2(K_2^2 - K_1) \end{pmatrix} = -Q$$

Finally, since (2.18) is the Lyapunov equation with Q is the definite positive matrix, and the result P is the definite positive symmetric matrix (even unique); therefore, the Φ must be the Hurwitz matrix. Hence, according to section 2.1.2, the equation (2.9) with the $\underline{e} = \underline{r} - \underline{q}$ must have the $\underline{e} \to \underline{0}$, and the $\underline{\dot{e}} \to \underline{0}$ at the same time (which needs to prove).

2.2.2 Evaluating the proposed controller robust performance for the uncertain bilinear Euler-Lagrange system

Documents [1] and [4] have presented an ISS robust control method for a general uncertain system (1.40), fully actuated, i.e., the system has the model:

$$M(\underline{q})\underline{\ddot{q}} + C(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + \underline{g}(\underline{q}) = \underline{u} + \underline{n}(t), \ \underline{q} \in R^n \qquad (2.19)$$

Based on the above results of [1], [4] on ISS stability, the dissertation comes up with a statement similar to the following, but now for the bilinear Euler-Lagrange class (2.11) has added uncertainty, is described:

$$M(\underline{q})\ddot{\underline{q}} + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}} = \underline{u} + \underline{n}(t) \tag{2.22}$$

Theorem 2: With two matrices K_1, K_2 :

$$K_1 = diag(k_{1i}), \ K_2 = diag(k_{2i})$$

$$k_{11} = \ \dots \ = k_{1n} = a, \ k_{21} = \ \dots \ = k_{2n} = \sqrt{ab} \tag{2.23}$$

have $b - 1 > a > 0$ *is optional, then the controller is:*

$$\underline{u} = M(\underline{q})\left[\ddot{\underline{r}} - \hat{\underline{d}}(t) + K_1\underline{e} + K_2\dot{\underline{e}} \right] + C(\underline{q},\dot{\underline{q}})\dot{\underline{q}} \tag{2.24}$$

where $\hat{\underline{d}}(t)$ *is the selected function, is satisfied:*

$$\left| \hat{\underline{d}}(t) - M(\underline{q})^{-1}\underline{n}(t) \right| \le \mu, \ \forall t, \underline{q} \tag{2.25}$$

will bring the tracking error $\underline{e} = \underline{r} - \underline{q}$ *of the bilinear Euler-Lagrange system containing the uncertainty (2.22), with its derivative* $\dot{\underline{e}}$, *towards the small around of the origin:*

$$\mathcal{O} = \left\{ \underline{p} = \text{col}(\underline{e},\dot{\underline{e}}) \in \mathbb{R}^{2n} \middle| \ |\underline{p}| \le \frac{\mu}{a} \right\} \tag{2.26}$$

Proof:

With two matrices K_1 and K_2 given in (2.23), the two matrices P and Q determined by formula (2.17), as the proof of theorem 1, are two positively defined symmetric matrices. Moreover, they also satisfy the Lyapunov equation (2.18) with the matrix Φ given in (2.8). In addition, the closed system, consisting of a bilinear Euler-Lagrange object (2.22) and the controller (2.24), is described:

$$\frac{d^2e}{dt^2} = -K_1\underline{e} - K_2\frac{d\underline{e}}{dt} + \hat{\underline{d}}(t) - M(\underline{q})^{-1}\underline{n}(t) \quad \text{or}$$

$$\frac{d\underline{p}}{dt} = \begin{pmatrix} \mathbf{0} & I \\ -K_1 & -K_2 \end{pmatrix}\underline{p} + \begin{pmatrix} \mathbf{0} \\ I \end{pmatrix}\underline{v} = \Phi\underline{p} + B\underline{v} \tag{2.27}$$

where:

$$\underline{p} = \begin{pmatrix} \underline{e} \\ \dot{\underline{e}} \end{pmatrix}, \ \underline{v}(t) = \hat{\underline{d}}(t) - M(\underline{q})^{-1}\underline{n}(t) \ and \ B = \begin{pmatrix} 0 \\ I \end{pmatrix}$$

Use a positive determination function:

$$V(\underline{p}) = \frac{1}{2}\underline{p}^T P \underline{p} \text{ with } \underline{p} = col(\underline{e}, \underline{\dot{e}}) = (p_1, p_2, \ \ldots \ , p_{2n})^T$$

together with (2.27), we have:

$$\frac{dV}{dt} = \frac{1}{2}\Big[(\Phi\underline{p} + B\underline{v})^T P \underline{p} + \underline{p}^T P(\Phi\underline{p} + B\underline{v}) \Big]$$

$$= \frac{1}{2}\Big[\underline{p}^T(\Phi^T P + P\Phi)\underline{p} + \underline{v}^T(B^T P + B^T P^T)\underline{p} \Big]$$

$$= \frac{1}{2}\Big(-\underline{p}^T Q \underline{p} + 2\underline{v}^T B^T P \underline{p}\Big)$$

$$= -\underline{p}^T \begin{pmatrix} K_1^2 & \mathbf{0} \\ \mathbf{0} & (K_2^2 - K_1) \end{pmatrix}\underline{p} + \underline{v}^T (\mathbf{0}, I)\begin{pmatrix} 2K_1 K_2 & K_1 \\ K_1 & K_2 \end{pmatrix}\underline{p}$$

$$= -\underline{p}^T \begin{pmatrix} K_1^2 & \mathbf{0} \\ \mathbf{0} & (K_2^2 - K_1) \end{pmatrix}\underline{p} + \underline{v}^T (K_1, K_2)\underline{p}$$

$$= -a^2 \sum_{i=1}^{n} p_i^2 - (ab - a)\sum_{i=1}^{n} p_{n+i}^2 + \big(ap_1, \ \ldots \ , ap_n, (ab - a)p_{n+1}, \ \ldots \ , (ab - a)p_{2n}\big)\underline{v}$$

$$\leq -a^2 |\underline{p}|^2 + a|\underline{v}||\underline{p}| \leq a\big(\mu - a|\underline{p}|\big)|\underline{p}|$$

Thus, when the tracking error $\underline{p}$ is far from the origin, it means $\underline{p} \notin \mathcal{O}$, then there is $\dot{V} < 0$, and so, $\underline{p}$ decreases or the tracking error is going to the origin, and that is the proof.

Finally, from theorem 2 we can see that when directly applying the ISS stable controller (2.24) to the uncertain Euler-Lagrange system (2.22), the bigger the chosen constant a, the gravitational domain $\mathcal{O}$ will be smaller. In practice, we can not choose $a = \infty$ so that the ISS stable tracking becomes asymptomatic stable tracking. Therefore, in order to control the asymptotic stable tracking, the remaining problem is how to choose the function vector $\hat{\underline{d}}(t)$ satisfying the requested condition (2.25) corresponding to $\mu = 0$. Dissertation will solve this problem in chapter 3.

2.2.3 Applying to TRMS and verifying controller performance by simulation on MatLab

The reference signals are the step and the sine wave:

$$\underline{r} = (\alpha_{hR}, \alpha_{vR})^T = (0.5, 0.3)^T \text{ [rad]}$$

$$\underline{r} = (\alpha_{hR}, \alpha_{vR})^T = (0.2\sin(0.1256t), -0.2\sin(0.1256t))^T \text{ [rad]}$$

Two matrices K_1, K_2, are chosen: $K_1 = \begin{pmatrix} 40 & 0 \\ 0 & 40 \end{pmatrix} \text{ and } K_2 = \begin{pmatrix} 30 & 0 \\ 0 & 30 \end{pmatrix}$
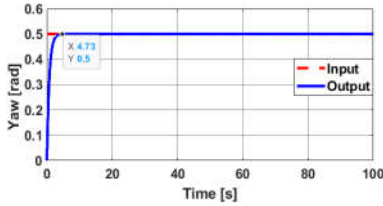
Figure 2.3: Step response of the yaw angle in the case of model without $\underline{g}(q)$
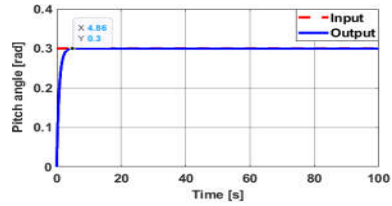

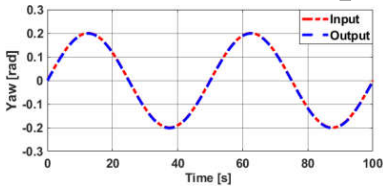*Figure 2.4: Step response of the pitch angle in the case of model without $\underline{g}(q)$*

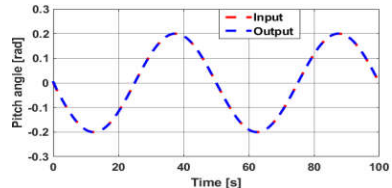*Figure 2.5: Sine wave response of the yaw angle in the case of model without $\underline{g}(q)$*

Figure 2.4: Sine wave response of the pitch angle in the case of model without $\underline{g}(q)$

2.3 Conclusions

The research results of the dissertation in this chapter include:

- Has briefly presented a traditional control method, which is the gravity compensation control method, as a basis for proposing the construction of an exact linearized controller of the dissertation, applied for the bilinear Euler-Lagrange system class, has fully actuated and does not contain uncertainty (called a system with exact model).

- Has been accurately demonstrated by theory (in Theorem 1) about the stable asymptotic tracking performance on the reference trajectories that this proposed controller gives the bilinear Euler-Lagrange system.

- In the case of the inexact bilinear Euler-Lagrange system, that is, itself exists disturbance, in this chapter, the dissertation has also discussed a proposed possibility to improve the controller to remain can achieve at least a stable ISS tracking performance (instead of asymptotic stabilization).

Although this additional result will not use to continue in the dissertation, it can be seen as a confirmation of the proposed controller's applicability for a wide class of the Euler-Lagrange systems when the model is inexact (with model deviations).

- In addition to the theoretical proof, the proposed controller's asymptotic tracking performance has also been demonstrated by the dissertation by simulation with a bilinear Euler-Lagrange TRMS, and without disturbance.

CHAPTER 3
UNCERTAINTY COMPENSATION CONTROL BASED ON TIME RECEDING OPTIMIZATION OF THE MODEL ERRORS

Nowadays, there are many methods to identify the general uncertainty nonlinear system, and therefore, they are also applicable to the bilinear system:

$$\dot{\underline{x}} = A(\underline{x})\underline{x} + B(\underline{x})\big[\underline{u} + \underline{d}(\underline{x}, t)\big] \qquad (3.1)$$

Where, $\underline{d}(\underline{x}, t)$ is a functional uncertainty, state dependent and having the same number of dimensions as the control signal, that is, the functional uncertainty component belongs to the image space $B(\underline{x})$.
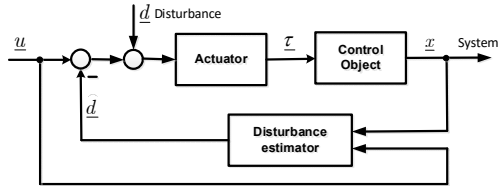
Hình 3.1: Structure of the compensation control system for input disturbance

It can be seen that after the uncertainties were identified and compensated as the described of Figure 3.1, the controller design (outer loop) will become simpler with only the control for the predetermined system described by bilinear model:

$$\dot{\underline{x}} = A(\underline{x})\underline{x} + B(\underline{x})\underline{u} \qquad (3.2)$$

so that its output $\underline{y} = \underline{g}(\underline{x})$ is asymptotically tracking with the preset reference trajectory $\underline{r}(t)$. That is also the work solved by the dissertation in chapter 2 for TRMS together **TRMS** control program

3.1 Disturbance identification algorithm
3.1.1 The disturbance system class having a bilinear state model

The dissertation limits the scope of the study for the class of nonlinear control objects, described by the bilinear model (including TRMS). When the actuator is assumed to be ideal, with having

$\underline{u} = \underline{\tau}$ (purely in terms of values), with no error ($\underline{d} = \underline{0}$) and no impact disturbance, the bilinear system in Figure 3.1 will have an ideal model is given in equation (3.2), where:

- $A(\underline{x})$, $B(\underline{x})$ are two state dependent matrices,

- $\underline{u}$, $\underline{x}$ are the vectors of control signals and the system state, respectively.

Thus, if we now consider the effect of input disturbance as well as actuator error, even when there is a model deviation with the assumption that the deviation belongs to the image space of the $B(\underline{x})$, the above idea model will become a functional uncertainty model of the general form:

$$\underline{\dot{x}} = A(\underline{x})\underline{x} + B(\underline{x})\left[\underline{u} + \underline{d}\right] \tag{3.3}$$

It can be seen that the uncertain bilinear model (3.3) is enough to cover a wide class of actual nonlinear systems, including uncertain Euler-Lagrange systems (2.1), fully actuated or underactuated [6], including TRMS, is the object chosen by the dissertation to simulate and evaluate the uncertain identification performance. Indeed, if we use the sign:

$$\underline{x} = \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} = \begin{pmatrix} \underline{q} \\ \underline{\dot{q}} \end{pmatrix}$$

then (2.1) becomes (3.3) with:

$$A(\underline{x}) = \begin{pmatrix} 0 & I \\ 0 & -M(\underline{q})^{-1}C(\underline{q},\underline{\dot{q}}) \end{pmatrix} \text{ and } B(\underline{x}) = \begin{pmatrix} 0 \\ M(\underline{q})^{-1}F \end{pmatrix} \tag{3.4}$$

3.1.2. Disturbance identification based on time receding minimization by model squared error

Figure 3.2 describes identifying the disturbance $\underline{d}(t)$ based on time receding minimization of the identification errors [69]:
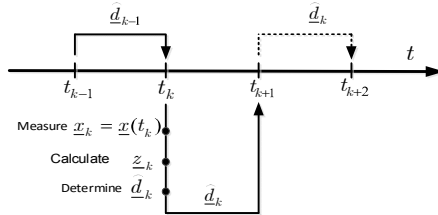
Figure 3.2: Principle of the disturbance identification based on time receding minimization

Disturbance identification algorithm (AD algorithm)

1. Choose an appropriate time receding horizon *Ts* at the time instant $t_k = kT_s$, $k = 0, 1, \ldots$ evenly spaced. These are the times at which $\underline{d}(t)$ will be approximately estimated as a series of

$\widehat{\underline{d}}_k \approx \underline{d}(t_k)$ values.

Here we need to assume that the matrix $B(\underline{x})$ of the bilinear system (3.3) is full rank at all points of state $\underline{x}_k = \underline{x}(t_k)$, that is, there is $rankB(\underline{x}_k) = n, \ \forall \underline{x}_k$.

Assign arbitrarily $\underline{z}_{-1}$ and $\widehat{\underline{d}}_{-1}$. Set $\underline{x}_{-1} = \underline{0}, \ k = 0$

2. Measure $\underline{x}_k = \underline{x}(t_k)$ from the controlled system and calculate:

$$\begin{bmatrix} A_k^x = I + T_s A(\underline{x}_{k-1}), \\ A_k^z = I + T_s A(\underline{z}_{k-1}), \\ B_k = T_s B(\underline{x}_{k-1}), \\ \underline{z}_k = A_k^z \underline{z}_{k-1} + B_k \underline{u}, \\ \widehat{\underline{d}}_k = \left[B_k^T B_k \right]^{-1} B_k^T \left(\underline{x}_k - \underline{z}_k + A_k^z \underline{z}_{k-1} - A_k^x \underline{x}_{k-1} \right) \end{bmatrix} \quad (3.15)$$

set $k := k + 1$ and turn back to step 2.

Regarding the performance of the disturbance identification, disturbance $\underline{d}(t)$ in the disturbance bilinear system (3.3) of the above identification algorithm, in [69] has confirmed and proved: "*If state $\underline{x}_k = \underline{x}(t_k)$ measured from the at time t_k accurately represented by the discontinuous model:*

$\underline{x}_k = A_k^x \underline{x}_{k-1} + B_k \left[\underline{u} + \underline{d}(t_k) \right]$ *where*

$A_k^x = I + T_s A(\underline{x}_{k-1})$ *and* $B_k = T_s B(\underline{x}_{k-1})$

*then, the estimated result obtained by **AD** algorithm will be accurate, this yield:* $\widehat{\underline{d}}_k = \underline{d}(t_k)$"

To speed up convergence for the above recognition algorithm, we can assign the variable: $\underline{z}_{k+1} = \underline{x}_{k+1}$

In such a case, the calculations (3.15) of the above given identity algorithm will be replaced by:

$$\left[\begin{array}{l} A_k^x = I + T_s A(\underline{x}_{k-1}), \\ A_k^z = I + T_s A(\underline{z}_{k-1}), \\ B_k = T_s B(\underline{x}_{k-1}), \\ \underline{z}_k = A_k^z \underline{z}_{k-1} + B_k \underline{u}, \\ \underline{\hat{d}}_k = \left[B_k^T B_k \right]^{-1} B_k^T \left(\underline{x}_k - \underline{z}_k + A_k^z \underline{z}_{k-1} - A_k^x \underline{x}_{k-1} \right) \\ \underline{z}_k = \underline{x}_k \end{array}\right. \tag{3.1 6}$$

3.2 Establishing a reference trajectory tracking adaptive controller for the disturbance bilinear Euler-Lagrange system

In chapter 2, the dissertation presents the asymptotic tracking control method for the bilinear Euler-Lagrange system with the exact model (2.11). Next, we will add to it the ability to disturbance identification and compensation control this disturbance identification, so that it is applicable to a class of the disturbance Euler-Lagrange system (2.19).

First, we convert the disturbance Euler-Lagrange model (2.19) to a bilinear disturbance Euler-Lagrange form by placing a new summarized system disturbance:

$$\underline{d}(\underline{q},t) = \underline{n}(t) - \underline{g}(\underline{q}) \tag{3.18}$$

Then, the initial disturbance Euler-Lagrange model (2.19) becomes: $M(\underline{q})\underline{\ddot{q}} + C(\underline{q},\underline{\dot{q}})\underline{\dot{q}} = \underline{u} + \underline{d}(\underline{q},t)$ (3.19)

Next, using the sign: $\underline{x} = \begin{pmatrix} \underline{q} \\ \underline{\dot{q}} \end{pmatrix}$ (3.20)

then model (3.19) becomes the disturbance bilinear state model:

$$\underline{\dot{x}} = \begin{pmatrix} 0 & I \\ 0 & -M(\underline{q})^{-1}C(\underline{q},\underline{\dot{q}}) \end{pmatrix}\underline{x} + \begin{pmatrix} 0 \\ M(\underline{q})^{-1} \end{pmatrix}\left[\underline{u} + \underline{d}(\underline{q},t)\right] =$$

$$= A(\underline{x})\underline{x} + B(\underline{x})\left[\underline{u} + \underline{d}(\underline{x},t)\right] \tag{3.21}$$

as shown in (2.32) and (3.4), where:

$A(\underline{x}) = \begin{pmatrix} 0 & I \\ 0 & -M(\underline{q})^{-1}C(\underline{q},\underline{\dot{q}}) \end{pmatrix}$, $B(\underline{x}) = \begin{pmatrix} 0 \\ M(\underline{q})^{-1} \end{pmatrix}$ and $\underline{d}(\underline{x},t) = \underline{d}(\underline{q},t)$, as

the model (3.3), suitable for the disturbance identification algorithm presented in section 3.1.2

3.2.1 Combining controller by tracking control and disturbance compensation

This combination control structure is shown in Figure 3.3 below:
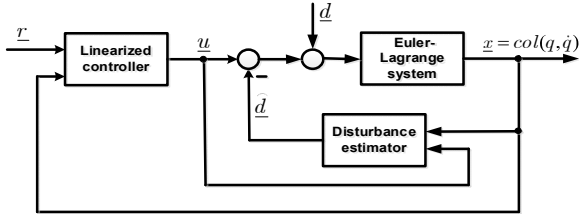
Hình 3.3: Exact linearized control combines
with disturbance compensation

3.2.2 Verifying performance control by simulation on MatLab with TRMS

- Step reference signals: $\underline{r} = \left(\alpha_{hR}, \alpha_{vR}\right)^T = \left(0.5, 0.3\right)^T$

- Sine wave reference signals:

$$\underline{r} = \left(\alpha_{hR}, \alpha_{vR}\right)^T = \left(0.2\sin(0.1256t), -0.2\sin(0.1256t)\right)^T$$

$$K_1 = \begin{pmatrix} 40 & 0 \\ 0 & 40 \end{pmatrix}; K_2 = \begin{pmatrix} 30 & 0 \\ 0 & 30 \end{pmatrix}$$

- Unknown function is assumed:

$$\underline{n}(t) = \begin{pmatrix} 0.04\sin(0.3t) + 0.02\cos(0.01t) \\ 0.08\cos(0.2t) + 0.07\sin(0.5t) \end{pmatrix}$$

, and then disturbance $\underline{d}(\underline{q}, t) = \underline{n}(t) - \underline{g}(\underline{q})$
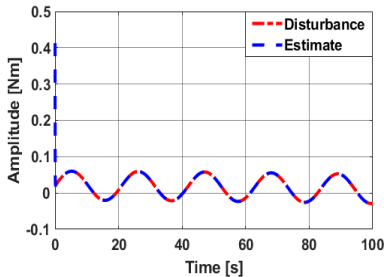
Figure 3.5: The result of the disturbance $\hat{d}_h(t)$ in the horizontal plane with the step reference signal
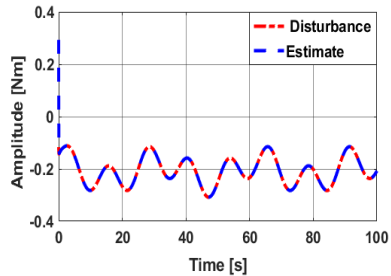
Figure 3.6: The result of the disturbanc $\hat{d}_v(t)$ e in the vertical plane with the step reference signal
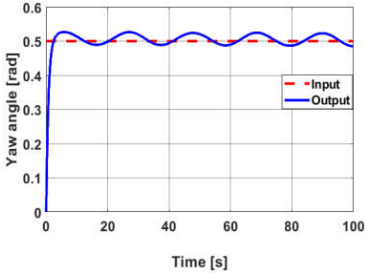
Figure 3.7: Step response of yaw angle without disturbance compensator
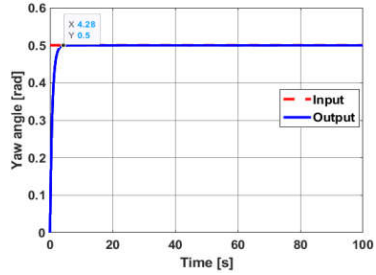
Figure 3.8: Step response of yaw angle with disturbance compensator
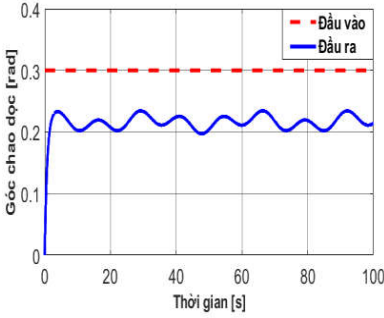
Figure 3.9: Step response of vertical angle without disturbance compensator
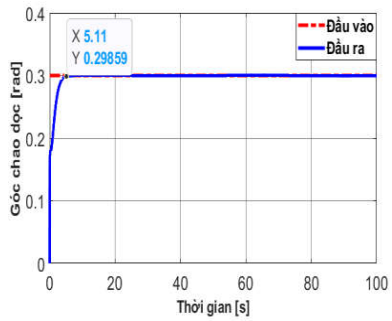
Figure 3.10: Step response of vertical angle with disturbance compensator
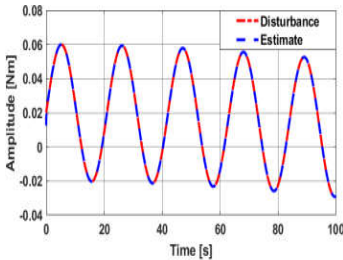
Figure 3.11: The result of the disturbance estimator $\hat{d}_h(t)$ in the horizontal plane with the sine wave reference signal
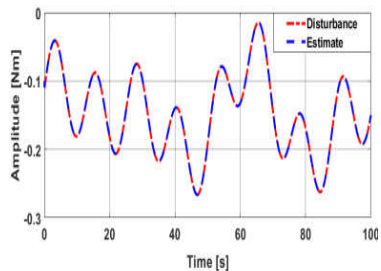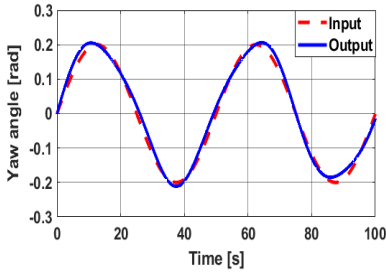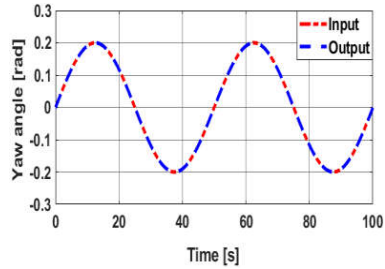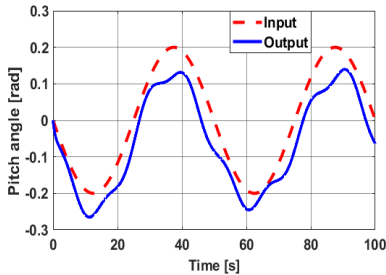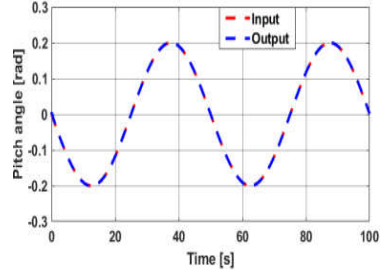

Figure 3.12: The result of the disturbance estimator $\hat{d}_v(t)$ in the horizontal plane with the sine wave reference signal

Figure 3.13: Sine wave response of yaw angle without disturbance compensator

Figure 3.14: Sine wave response of yaw angle with disturbance compensat

Figure 3.15: Sine wave response of vertical angle without disturbance compensator

Figure 3.16: Sine wave response of pitch angle with disturbance compensator

3.3. Conclusions

In chapter 3, the dissertation has achieved the following research results:

- Establishing an algorithm to estimate the uncertainty of the bilinear Euler-Lagrange model, serving for compensatory control.

- Combining the disturbance compensation controller with the exact linearized controller in chapter 2 to get the reference signal asymptotic tracking stable controller for bilinear Euler-Lagrange systems.

- Evaluating the proposed controller performance by simulations on MatLab for TRMS

CHAPTER 4
IMPLEMENTATION AND EXPERIMENTAL RESULTS

4.1 Laboratory table description

This is the experimental table available at the Research development Institute of Advanced Industrial Technology of Thai Nguyen University of Technology.

4.1.1 The devices on the laboratory table
4.1.1.1 Computer
4.1.1.2 Card dSPACE DS 1103
4.1.1.3 TRMS physical system
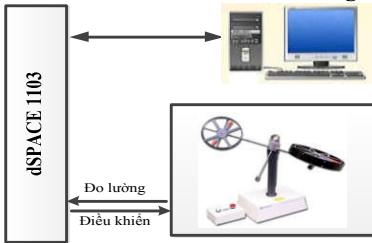*4.1.1.4 External disturbance generation system*

Figure 4.1: Physical structure of TRMS experimental table

Figure 4.5: Model of TRMS experimental table

4.1.2 Overall structure of TRMS testing table of Thai Nguyen University of Technology

Figure 4.5 is the Model of TRMS experimental table, and Figure 4.14 is structure of TRMS real system designed on Simulink.
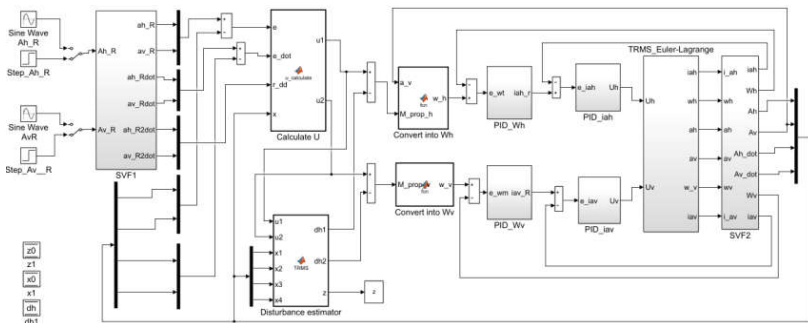
Figure 4.14: Structure of TRMS real system designed on Simulink

4.3 Experimental results and evaluation
4.3.1 Performing experiments

The reference signals are the step:

$$\alpha_h \to r_1 = 0.5\,[rad]\ \text{and}\ \alpha_v \to r_2 = 0.3\,[rad]$$

The reference signals are the sine wave:

$$\alpha_h \to r_1 = 0.2\sin(0.1256t)\,[rad];\ \alpha_v \to r_2 = -0.2\sin(0.1256t)\,[rad]$$

Exact linearized controller, of which two matrices K_1, K_2 are still selected respectively:
$$K_1 = \begin{pmatrix} 40 & 0 \\ 0 & 40 \end{pmatrix};\ K_2 = \begin{pmatrix} 30 & 0 \\ 0 & 30 \end{pmatrix}$$

- Speed and current controller parameters for the tail motor:

$$\text{PID}_{\omega_h} : K_{p,\omega_h} = 0.00001,\ K_{I,\omega_h} = 0.000005,\ K_{d,\omega_h} = 0.000035$$

$$\text{PID}_{i_{ah}} : K_{p,i_{ah}} = 0.25,\ K_{I,i_{ah}} = 20,\ K_{d,i_{ah}} = 0.0028$$

- Speed and current controller parameters for the tail motor:

$$\text{PID}_{\omega_v} : K_{p,\omega_v} = 0.000004,\ K_{I,\omega_v} = 0.00005,\ K_{d,\omega_v} = 0.000015$$

$$\text{PID}_{i_{av}} : K_{p,i_m} = 0.1,\ K_{I,i_{av}} = 80,\ K_{d,i_{av}} = 0.0005$$

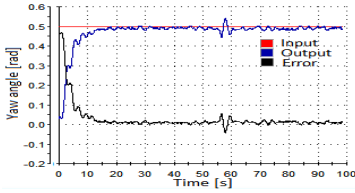### 4.3.2 Results and performance assessment

Figure 4.25: Step response of yaw angle and error under a noise of blow fan at $t = (50 \div 100)s$
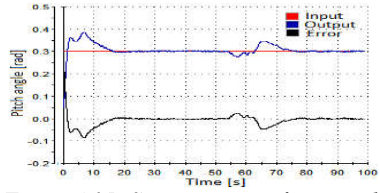

Figure 4.25: Step response of yaw angle and error under a noise of blow fan at $t = (50 \div 100)s$
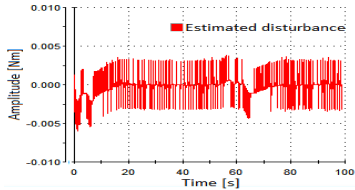

Figure 4.27: The estimated disturbance $\hat{d}_h(t)$ on the horizontal plane corresponds to the step reference under the noise of blow fan at $t = (50 \div 100)s$
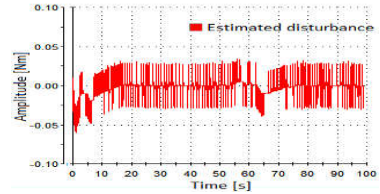

Figure 4.28: The estimated disturbance $\hat{d}_v(t)$ on the vertical plane corresponds to the step reference under the noise of blow fan at $t = (50 \div 100)s$
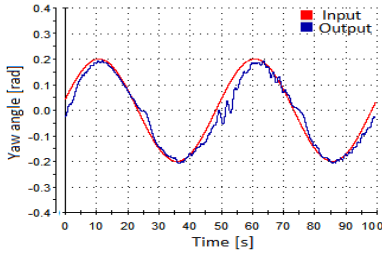
Figure 4.29: Sine wave response of yaw angle and error under a noise of blow fan at $t = (50 \div 100)s$
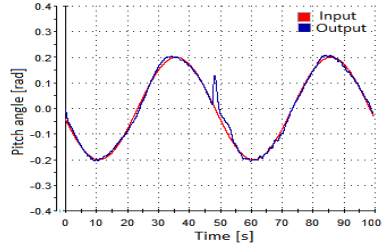

Figure 4.30: Sine wave response of pitch angle and error under a noise of blow fan at $t = (50 \div 100)s$
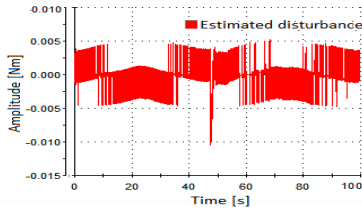

Figure 4.33: The estimated disturbance $\hat{d}_h(t)$ on the horizontal plane corresponds to the sine wave reference under the noise of blow fan noise at $t = (50 \div 100)s$
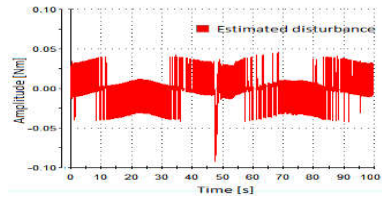

Figure 4.34: The estimated disturbance $d_v(t)$ on the horizontal plane corresponds to the sine wave reference under the noise of blow fan noise at $t = (50 \div 100)s$

4.4 Conclusions

In chapter 4, the following tasks are solved:

- The designed, installed, set up, and tested with the TRMS in the laboratory with an exact linearized controller combined with the disturbance estimator to control disturbance compensation. This exact linearized controller is torque controller so to test the system needs two additional control loop: the speed and the current control loops.

- The test results on the TRMS experimental table are matched with the theory of control performance confirmed in chapters 2 and 3. These results have also demonstrated the correctness of the disturbance estimation and disturbance compensation algorithm when the object model is uncertain and affected by disturbance, and has enhanced disturbance resistance to improve the adaptive control performance of the system.

CONCLUSIONS AND RECOMMENDATIONS

I. CONCLUSIONS

The results of the dissertation research have made some new contributions as follows:

- Establishing the trajectory tracking controller for the bilinear electromechanical system, proving the stability and asymptotic stability of the tracking system in the absent disturbance and disturbance. Based on the traditional control method, which is the gravitational compensation control, as the basis for proposing the establishment of an exact linearized controller of the dissertation, applying for the bilinear Euler-Lagrange system class, have fully actuators, and no disturbance (called an exact modeling system). Strictly demonstrated by theory (in the theorem 1 and 2) about the stable tracking performance according to the reference trajectory that this proposed controller gives the bilinear Euler-Lagrange system;

- Establishing the trajectory tracking adaptive controller for the mechatronic bilinear system on the basis of the linearized controller and the disturbance identifier based on time receding minimization by model squared error. Based on established the disturbance estimator algorithm in the bilinear Euler-Lagrange model for doing compensatory control. Combine with the disturbance controller with the exact linearized controller in Chapter 2 to obtain a reference signal asymptotic stable tracking controller b for bilinear Euler-Lagrange systems.

II. RECOMMENDATIONS

- Studying the TRMS model, which can apply to different controllers, in training at graduate and post-graduate level of the university;

- With results of this study can provide to fly elements with complex aerodynamic forms such as UAVs and other nonlinear control methods applied to TRMS.